

# Some Sharp upper Bound Estimates for the Maximal Modulus of Polar Derivative of a Polynomial

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**Abstract**—*In this paper, we establish some sharp upper bound estimates for the maximal modulus of polar derivative of a complex polynomial on a disk under the assumption that the polynomial has no zeros in another disk. The obtained results sharpen some already known estimates for the ordinary derivative of polynomials as special cases.*

**Keywords:** *Polar derivative, Bernstein inequality, Maximal modulus, Zeros.*

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